

## Laboratory Measurements

Three students measure the length of an erasure with a ruler. The measurements they report are 12 cm, 12.3 cm, 12.38 cm, and 12.375 cm.

*If three people measure the same thing will all three measurements have the same number of significant figures?*

In this class we will **all** read measurements to the same precision!

*Are some measuring devices more accurate than others?*

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*If three people measure the same thing will the measurements be the same?*

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Steps to producing reproducible measurements:

In order to maintain the same precision we will all read our meters, rulers and other analog devices the same way. Every measurement must include the value that is certain, an estimate of the closeness that the measurement is to the certain value, an indicator of the uncertainty of the measurement, and the unit of the measurement. We will use the symbols, N, m, M, u and U in our discussion.

**N:** The measured value is at least this numbered mark (N) on the scale. (This value is certain)

**m \* M:** The number of marks (m) past N multiplied by the value of each mark, M. (This value is also certain.)

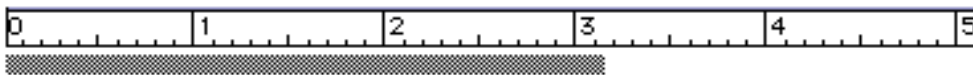
**u \* U:** The number of uncertain steps (u) past the last certain mark multiplied by the value of the uncertainty (U). (This value is uncertain.)

A measurement -- sums these parts:  $N + (m * M) + (u * U)$

-- includes the uncertainty :  $\pm U$

-- includes the **unit** of measurement (mL, g, cm...)

example: ( a measurement of length in cm)



1. The measured value is at least which numbered mark (N) on the scale?

*Example:* The measured value is greater than 3 cm.  $N = 3$  cm

2. Determine the value of each division or mark of the scale (M):

Choose a range (two adjacent numbers) and subtract the values.

*Example:* Range = 4 cm - 3 cm = 1 cm

Determine the number of divisions in the range by counting the spaces between the two numbers.

*Example:* Divisions = 10 spaces between 3 and 4 cm

Divide the range by the number of divisions - this is the value of each division or mark.

*Example:* Value of mark = Range/Divisions = 1/10 = 0.1 cm

The value of the mark or division is certain (known with confidence).

*Example:* Each of the small markings between 3 and 4 represent 0.1 cm.

**M** = 0.1 cm

3. Determine the number of marks past N. This value is **m**.

*Example:* The measurement is 1 line beyond the 3 cm mark. **m** = 1

4. Record the certain part of the measurement: **N + (m \* M)**

*Example:* 3 cm + 1 \* 0.1 cm = 3.1 cm

The actual measurement is at least 3.1 cm.

But each measurement must also include an estimate of how close the actual measurement is to the certain part of the measurement. In fact, we can see that the line we are measuring goes past 3.1 cm. We will estimate to the nearest 10% of each line. This is done as follows:

5. Determine the uncertainty (**U**) of the scale:

Divide the mark value (**M**) by 10 to represent 10 imaginary divisions in the space between the lines.

*Example:*  $U = M/10 = 1 \text{ cm} / 10 = 0.01 \text{ cm}$

This tells you how many decimal places to record this measurement.

*Example:* A measurement using this scale will have two decimal places.

6. The measurement lies how many uncertain steps (**u**) beyond the last mark (**N + m \* M**)?

*Example:* The indicator is just about halfway (5 out of 10 possible steps) so **u** = 5.

7. Determine the uncertain part of the measurement: **u \* U**

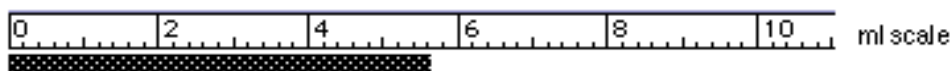
This part of the measurement is uncertain since two individuals may estimate **u** differently.

*Example:*  $u * U \pm U = 5 * 0.01 = 0.05$

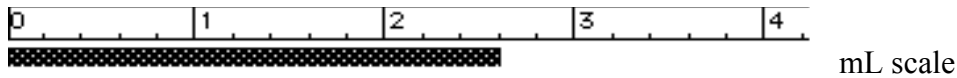
8. Determine the measurement by adding the certain (**N + m \* M**) and uncertain (**u \* U**) components together.

*Example:*  $N + (m * M) + (u * U) = 3 + 1*0.1 + 5*0.01 = 3.15 \text{ cm}$

### Example 2



N 4 m 8 M 0.2 u 2 U 0.02 so:  $4 + (8 \cdot .2) + (2 \cdot .02) = 5.64$  ml  
**Practice (Your turn!)**



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

## UNCERTAINTIES

*How good are the measurements we take?*

In the previous discussion we saw that we had certain parts (N+mM) and an uncertain part (uU). In example 2, the measurement was 5.64 ml. How certain can we be about the value of small u. We choose 2 but someone else may see 3 or 1. If we look strictly at the errors due to the reading of the scale, we could say that there is a very good chance that the true reading is  $\pm U$  or  $5.64 \pm .02$  ml. There are other errors that usually may occur when we take measurements. Take a few minutes and think of some problems that can cause deviations from a good reading. Do these errors always change the value a certain amount in one direction or can it affect the reading varying amounts in different directions.

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

*How do we identify how these can effect our measurements? How large are these errors?*

How can we identify the magnitude of these errors. In general, we take a number (the more the better) of measurements and then find the average. We assume that the average is the “correct answer”. Is this a correct assumption? Why?

If we look at how widely spaced the measurements are we can get an idea of how large the errors are. Statistics tell us that if we can define something called a standard deviation. We will report our measurements as  $\pm$  one standard deviation. Statistics tells us that there is a 68.5% chance that the correct value is the average  $\pm$  a standard deviation. . Statistics tells us that there is a 94.5% chance that the correct value is the average  $\pm$  two standard deviations.

Lets take a look at some data from a Baush and Lomb Spectronic 20 visible spectrometer.

Trial	Reading
1	1.67
2	1.63
3	<u>1.70</u>
Avg.	1.67 ppm    This number is our best estimate of the “correct answer”

The standard deviation is defined as

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i - x_{ave})^2}{N-1}} = \sqrt{\frac{\sum_{i=1}^N x_i^2 - (\sum_{i=1}^N x_i)^2 / N}{N-1}}$$

The right hand equation is easier to use. This is the equation for a less than 20 pieces of data. For more than 20 data points we use the same equation but with N in the denominator instead of N-1. Lets find the standard deviation for this set of three data points.

$x_i$	$(x_i)^2$
1.67	2.7889
1.63	2.6569
1.70	2.8900
$\sum x_i = 5.00$	$\sum (x_i)^2 = 8.3359$

$$s = \sqrt{\frac{8.3358 - (5.00)^2 / 3}{3-1}} = 0.0351 = .04 ppm$$

We can also do this on our calculator.

Type in the three numbers into list 1. Press (Stat) then go to (Calc) and then 1-Var Statistics.

$\bar{x}$  is clearly the average.

Sx is the standard deviation we use for small groups of data.

$\sigma_x$  is the standard deviation for large groups of data (greater than 20 data points)

If the standard deviation is small does that mean that  $\bar{x}$  is the correct answer?

What is the definition of accurate?

What is the definition of precise?

## ERROR PROPAGATION

When measurements are used in a calculation the result of the calculation should have an indication of the error of those measurements. If several measurements are involved, then the error in the calculation result must reflect the errors in the individual measurements. This is called error propagation. Remember that in this class, the standard deviation will be the error.

### ADDITION/SUBTRACTION

The error is additive, whenever the operation involved is addition or subtraction. For example, if the sum of three masses is needed, then the error of the sum is the sum of the errors of the measurements.

$$(2.01 \pm .01 \text{ g}) + (2.05 \pm .01 \text{ g}) + (1.98 \pm .01 \text{ g}) = 6.04 \pm .03 \text{ g}$$

The average mass and the error of the average is found by dividing by a constant, 3.

Average mass:  $2.01 \pm .01 \text{ g}$

A student is measuring the mass of a liquid in a graduated cylinder by measuring the dry cylinder first, measuring the cylinder and the liquid together, and then subtracting the result.

$$48.35 \pm .01 \text{ g} - 42.43 \pm .01 \text{ g} = 5.92 \pm .02 \text{ g}$$

Why is the student less certain of the result of the subtraction than he is of each individual measurement? The dry graduated cylinder has a mass of 42.42 to 42.44 g, but isn't known exactly. The combined mass of the liquid and the cylinder is between 48.34 and 48.36 g, and isn't exactly known either. If the correct masses are the measurements furthest apart (48.36 g and 42.42 g), the mass of the liquid is 5.94 g. If the correct masses are the measurements closest together (48.34 and 42.44), the mass of the liquid is 5.90 g. The actual mass of the liquid must be between 5.90 g and 5.94 g. Therefore, we record the mass as  $5.92 \pm .02 \text{ g}$ !

### MULTIPLICATION AND DIVISION

When multiplying and dividing measurements the error propagation formula is a little more challenging. For example we are multiplying or dividing two measurements A and B. The value for this calculation is V. The uncertainty for this calculation is given below:

$$U=V\sqrt{\left(\frac{U_A}{A}\right)^2+\left(\frac{U_B}{B}\right)^2}$$

where  $U$  is the error of the calculation result,  $U_A$  is the error of measurement A and  $U_B$  is the error of measurement B.

For example, calculate the density of a solid that has a mass of  $19.23 \pm .01$  g and a volume of  $4.2 \pm .1$  cm<sup>3</sup>.

$$m = 19.23 \pm .01 \text{ g} \quad V = 4.2 \pm .1 \text{ cm}^3 \quad d = m/V = (19.23 \text{ g})/(4.2 \text{ cm}^3) = 4.57857 \text{ g/cm}^3$$

To determine the error, or uncertainty, of the density, use the error propagation formula. Use the mass (19.23 g) and its uncertainty (.01 g) as the values of A and  $U_A$  and use the volume (4.2 cm<sup>3</sup>) and its uncertainty (.1 cm<sup>3</sup>) as the values of B and  $U_B$ .

The uncertainty of the density should have the same number of decimal places as the measurement. Since the measurements 19.23 g and 4.2 mL allow only 2 significant figures in the density, record the density as  $4.6 \pm .1$  g/cm<sup>3</sup>.

Let try a density example of our own. Here are 6 mass measurements:

12.34g , 12.40g, 12.31g, 12.28g, 12. 37g, 12.50g.

The average is \_\_\_\_\_ (A)and the standard deviation is \_\_\_\_\_( $U_A$ ).

Here are 6 volume measurements:

2.34 ml, 2.30ml, 2.50ml, 2.36ml, 2.36ml, 2.40ml.

The average is \_\_\_\_\_ (B)and the standard deviation is \_\_\_\_\_( $U_B$ )..

Plug this into the equation  $U = V \sqrt{\left(\frac{U_A}{A}\right)^2 + \left(\frac{U_B}{B}\right)^2}$ .

Note that some numbers are not measurements and therefor and zero uncertainty, There are **exactly** 100 cm in one meter. Because that number is defined there is no uncertainty. Numbers from counting are also considered to have no uncertainty. "There are exactly 21 people in this lab". Because there cannot be a half a person we must assume that the number is exact and has no uncertainty.

**MEASUREMENT PRACTICE**

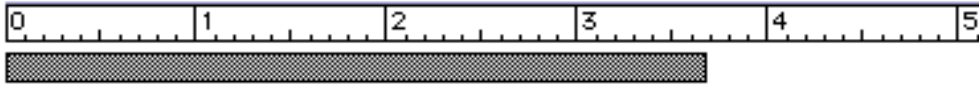
View the measurement videos on the web.

<http://web.fccj.org/~ksanchez/flash/Measurements/measurements.htm>

Record the

measurements:

1. gram scale



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

measurement: \_\_\_\_\_

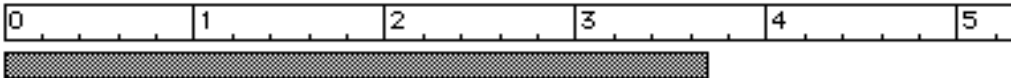
2. centimeter scale



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

measurement: \_\_\_\_\_

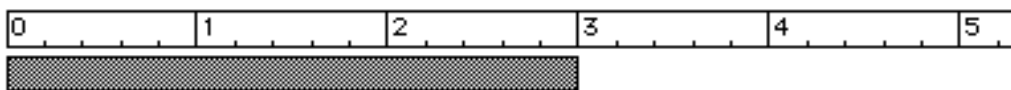
3. milliliter scale



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

measurement: \_\_\_\_\_

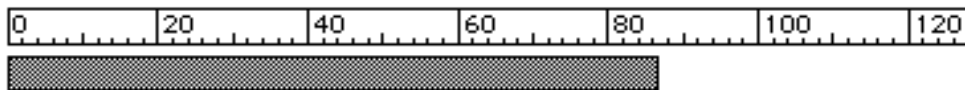
4. milliliter scale



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

measurement: \_\_\_\_\_

5. \_centigrade scale



N \_\_\_\_\_ m \_\_\_\_\_ M \_\_\_\_\_ u \_\_\_\_\_ U \_\_\_\_\_

measurement: \_\_\_\_\_